

سیستمی نظم

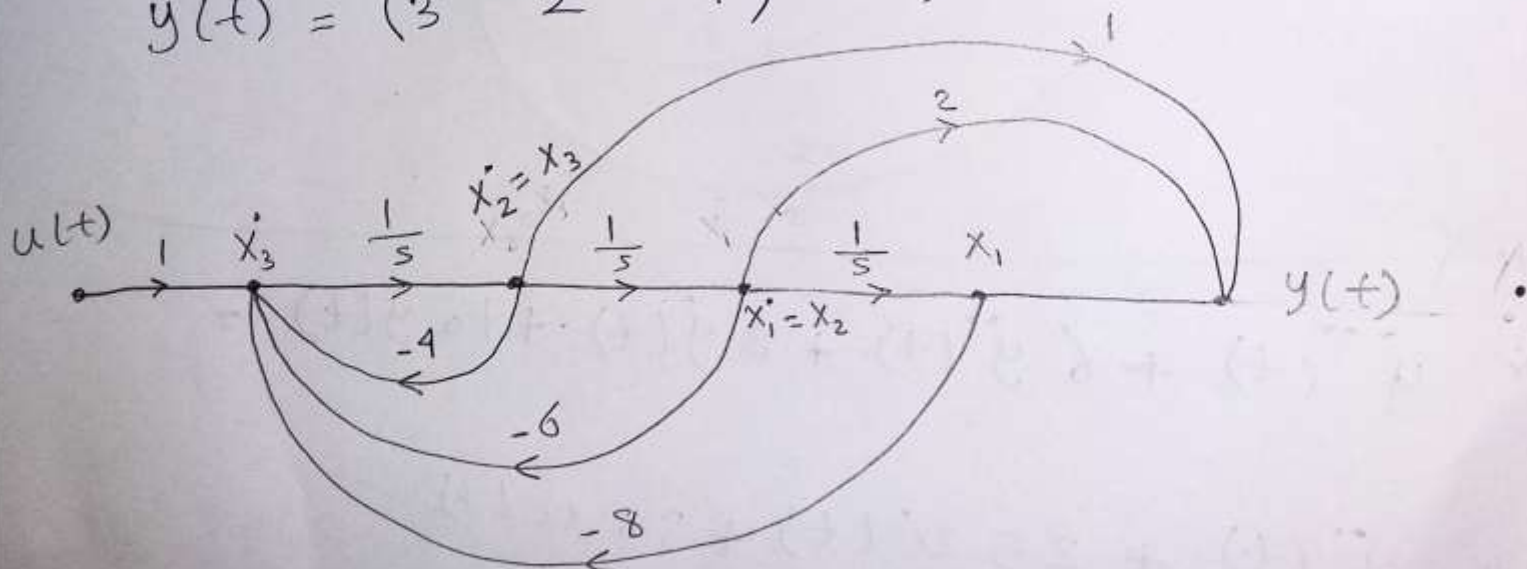
$$c) \frac{Y(s)}{U(s)} = \frac{s^2 + 2s + 3}{s^3 + 4s^2 + 6s + 8}$$

$$\dot{X}(t) = \underset{3 \times 3}{A} \underset{3 \times 1}{X(t)} + \underset{3 \times 1}{B} u(t)$$

$$y(t) = \underset{1 \times 3}{C} X(t) + D u(t)$$

$$\dot{X}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

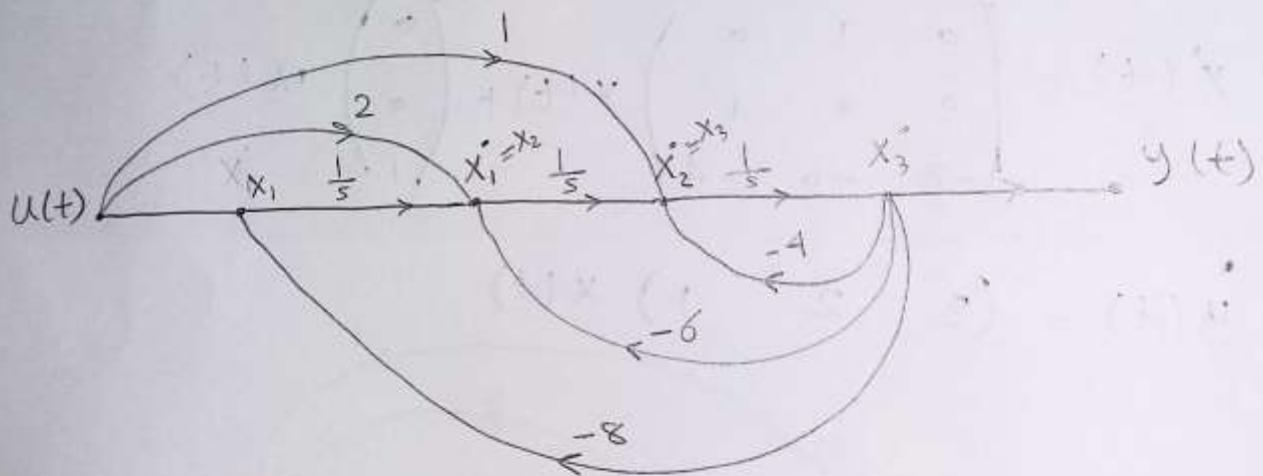
$$y(t) = (3 \quad 2 \quad 1) X(t)$$



1 Last sec.

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{pmatrix} 0 & 0 & -8 \\ 1 & 0 & -6 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$



$$* \quad y'''(t) + 6y''(t) + 8y'(t) + 10y(t) =$$

$$20\ddot{u}(t) + 25\dot{u}(t) + 30u(t)$$

[2] Last sec.

$$* \ddot{y}'''(t) + 6 \ddot{y}''(t) + 8 \dot{y}'(t) + 10 y(t) = 20 \ddot{u}''(t) + 25 \dot{u}(t) + 30 u(t)$$

Laplace

$$s^3 Y(s) + 6s^2 Y(s) + 8s Y(s) + 10 Y(s) = 20s^2 U(s) + 25s U(s) + 30 U(s)$$

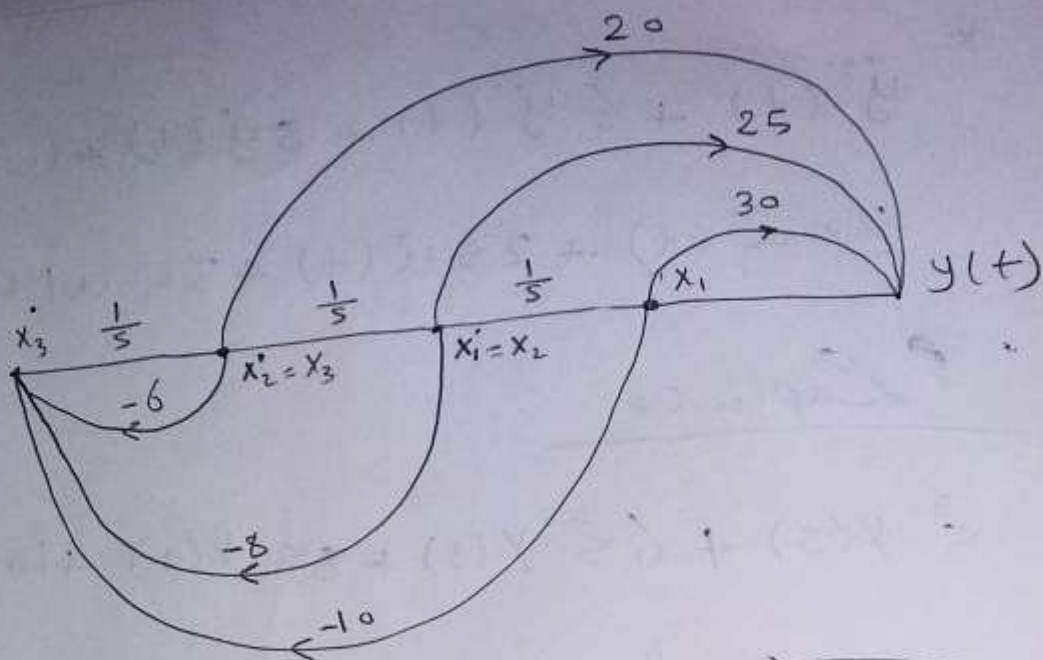
$$Y(s) [s^3 + 6s^2 + 8s + 10] = U(s) [20s^2 + 25s + 30]$$

$$\frac{Y(s)}{U(s)} = \frac{20s^2 + 25s + 30}{s^3 + 6s^2 + 8s + 10}$$

$$y(t) = \begin{pmatrix} 30 & 25 & 20 \end{pmatrix} x(t)$$

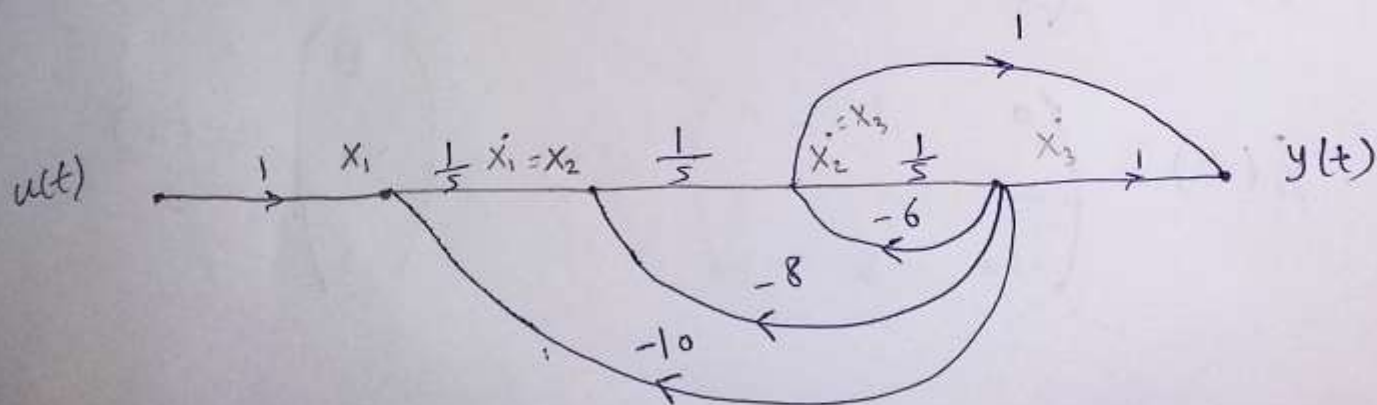
$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -8 & -6 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

3 Last sec.



$$\dot{x}(t) = \begin{pmatrix} 0 & 0 & -10 \\ 1 & 0 & -8 \\ 0 & 1 & -6 \end{pmatrix} x(t) + \begin{pmatrix} 30 \\ 25 \\ 20 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t)$$



[4] Last section

$$\boxed{2} \quad \dot{x}(t) = \underbrace{\begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix}}_A x(t) + \underbrace{\begin{pmatrix} 2 \\ 4 \end{pmatrix}}_B u(t)$$

$$y(t) = \underbrace{(1 \quad 2)}_C x(t)$$

Sol

$$\frac{X(s)}{U(s)} = C (sI - A)^{-1} B + D$$

$$\left[s \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \right]^{-1} = \left[\begin{pmatrix} s+5 & 1 \\ -3 & s+1 \end{pmatrix} \right]^{-1}$$

مصفوفة الوحدة

$$= \frac{1}{(s+5)(s+1) - 3} \begin{pmatrix} s+1 & -1 \\ 3 & s+5 \end{pmatrix}$$

$$\frac{(1 \quad 2)}{s^2 + 6s + 8} \begin{pmatrix} s+1 & -1 \\ 3 & s+5 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{Y(s)}{U(s)}$$

$$= \frac{1}{s^2 + 6s + 8} (1 \quad 2) \begin{pmatrix} 2s-2 \\ 4s+26 \end{pmatrix}$$

5 Last sec.

$$\frac{X(s)}{U(s)} = \frac{Y(s)}{U(s)} = \frac{2s - 24 + 8s + 52}{s^2 + 6s + 8}$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{10s + 50}{s^2 + 6s + 8}}$$

$$\boxed{4} \quad \dot{x}(t) = \begin{pmatrix} 0 & 1 \\ +3 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad u(t) \rightarrow \text{unit step}$$

$$x(t) = \phi(t) \cdot x(0) + \int_0^t \phi(t-\tau) B u(\tau) d\tau$$

$$\phi(t) = \mathcal{L}^{-1} \left[\frac{\boxed{\text{sol}}}{(sI - A)} \right]$$

$$\phi(t) = \mathcal{L}^{-1} \left(\begin{pmatrix} s & -1 \\ -3 & s+2 \end{pmatrix}^{-1} \right)$$

$$= \mathcal{L}^{-1} \left[\frac{1}{s^2 + 2s - 3} \cdot \begin{pmatrix} s+2 & 1 \\ +3 & s \end{pmatrix} \right]$$


Last. sec $\boxed{6}$

$$= \mathcal{L}^{-1} \left(\begin{array}{cc} \frac{s+2}{s^2+2s-3} & \frac{1}{s^2+2s-3} \\ \frac{-3}{s^2+2s-3} & \frac{s}{s^2+2s-3} \end{array} \right)$$

$$= \mathcal{L}^{-1} \left(\begin{array}{cc} \frac{\frac{1}{2}}{s+3} + \frac{\frac{3}{4}}{s-1} & \frac{A_3}{s+3} + \frac{A_4}{s-1} \\ \frac{A_1}{s+3} + \frac{A_2}{s-1} & \frac{A_5}{s+3} + \frac{A_6}{s-1} \end{array} \right)$$

$$= \begin{pmatrix} \frac{1}{2} e^{-3t} + \frac{3}{4} e^t & A_3 e^{-3t} + A_4 e^t \\ A_1 e^{-3t} + A_2 e^t & A_5 e^{-3t} + A_6 e^t \end{pmatrix}$$

$$\phi(t) * x(0) = \begin{pmatrix} B_1 e^{-3t} + B_2 e^t \\ B_3 e^{-3t} + B_4 e^t \end{pmatrix}$$

التوابت دي تسارى
أرقام  لاتكتب
كتوابت يعني .

$$\int_0^t \begin{pmatrix} A_3 e^{-3-\tau} + A_4 e^{(t-\tau)} \\ A_5 e^{-3-\tau} + A_6 e^{t-\tau} \end{pmatrix} d\tau$$

(2)

7 Last sec.

$$= \begin{pmatrix} \frac{A_3}{3} - \frac{A_3}{3} e^{-3t} + \frac{A_4}{3} + A_4 e^t \\ \frac{A_5}{3} - \frac{A_5}{3} e^{-3t} - A_6 + A_6 e^t \end{pmatrix} \rightarrow (2)$$

$$x(t) = (1) + (2)$$

$$\boxed{6} \quad \dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -8 & -6 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 8 & 1 \end{pmatrix} x(t)$$

$$\text{Sol} \quad \det(sI - A) = \begin{vmatrix} s & -1 \\ 8 & s+6 \end{vmatrix}$$

$$= s^2 + 6s + 8 = 0$$

$$(s + 2)(s + 4) = 0 \rightarrow s = -2, s = -4$$

on L.H.s \rightarrow system stable.

$\boxed{8}$ Last sec.

$$\frac{Y(s)}{U(s)} = \begin{pmatrix} 8 & 1 \end{pmatrix} \begin{pmatrix} s+6 & 1 \\ -8 & s \end{pmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} * \frac{1}{(s+2)(s+4)}$$

$$= \begin{pmatrix} 8 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ s \end{pmatrix} * \frac{1}{(s+2)(s+4)}$$

$$= \frac{s+8}{(s+2)(s+4)}$$

$$\phi(t) = \mathcal{L}^{-1} (sI - A)^{-1}$$

$$\underline{ds_i} :$$

\boxed{g} Last sec.